# A Hybrid Quantum Encoding Algorithm of Vector Quantization for Image Compression

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## Abstract

Many classical encoding algorithms of Vector Quantization (VQ) of image compression that can obtain global optimal solution have computational complexity $O(N)$. A pure quantum VQ encoding algorithm with probability of success near 100% has been proposed , that performs operations $45\sqrt{N}$ times approximately. In this paper, a hybrid quantum VQ encoding algorithm between classical method and quantum algorithm is presented. The number of its operations is less than $\sqrt{N}$ for most images, and it is more efficient than the pure quantum algorithm.

Keywords: Vector Quantization, Grover's Algorithm, Image Compression, Quantum Algorithm



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## I. INTRODUCTION

The two open research problem of vector quantization (VQ) are to design real-time encoding algorithm and to design the codebook generation algorithm that can generate approximately local optimal solution (finding global optimal solution is NPC problem) [1]. The property of local clustering is revealed by Pang [1] and it is used to design fast codebook generation algorithm, that is faster than the famous LBG algorithm by a factor of 4~13 [2]. But the other open problem, i.e., fast encoding of VQ with global optimal solution has not been solved. The encoding is described as [1]:

Let codebook

$$C = \{c[i] \mid 0 \leq i < N, c[i] \in R^k, k \geq 1\}$$

For an arbitrary input vector $x \in R^k$ , we have to find the index $i_0$ such that distance $d(x, c[i_0])$ is minimum, where $R^k$ is $k-$ dimensional Euclidean Space. Vector $c[i]$ is called codevector.

Grover's algorithm [3] is famous quantum search algorithm, and its accuracy of rotation is improved recently [4]. G.-L. Long proposes a modified Grover's algorithm [5]. Long's algorithm has the probability of success 100% even for the case that the size of database $N$ is very small. This property of Long's algorithm is suitable for quantum image compression because $N$ is not a giant number in image compression in general. Some hybrid search algorithms have also been proposed to improve efficiency of algorithms [6, 7], in which more physical resources are used to commute the efficiency.

Boyer, Brassard, H$\phi$yer and Tap propose an iteration algorithm named BBHT algorithm in this paper, in which the Grover iteration is applied repeatedly [8]. BBHT algorithm (Boyer, Brassard, H$\phi$yer, Tap) [8] is designed specially to solve the searching case that the number of solutions is unknown.

Quantum image compression is possible [9, 10, 11]. Pang et.al., present a Quantum Discrete Cosine Transformation (QDCT) for image compression [9]. The method of QDCT can be applied to other transformation such as Fourier transformation and classical output will be brought. Latorre presents a nice and maybe hopeful quantum image compression method. Latorre's algorithm requires that quantization method (scalar quantization or VQ) is incorporated with it. Otherwise, it is not competitive with classical methods [10]. Pang, et.al., present a pure quantum VQ encoding algorithm with probability of success near100%, that performs operations $45\sqrt{N}$ times approximately [11].
This paper presents a hybrid quantum VQ algorithm between classical method and quantum algorithm to solve the *open problem* of fast encoding of VQ. And the number of its operations is less than $\sqrt{N}$ and its probability of success is 100% approximately.

**Notation 1** *$x$: input vector*

**Notation 2** $\delta_0 = \min\{d(c[i], c[j]) \mid i \neq j, 0 \leq i, j < N\}$

**Notation 3** *$I$: whole encoding space*

**Notation 4** $S = \{x \mid d(x, c[i_0]) < \frac{\delta_0}{2}\}$ *and* $T = \{x \mid d(x, c[i_0]) < \hat{\delta}\}$, *where* $\hat{\delta} \geq \frac{\delta_0}{2}$.

**Notation 5** $\Omega(c[i]) = \{c[j] \mid d(c[i], c[j]) < 2\hat{\delta}, c[j] \in C\}$

**Notation 6** $Inf_{\Omega} = \min\{\mid \Omega[c[i]] \mid \mid 0 \le i < N\}$

All of the notations are diagrammatized in figure 2.

Parameter $\delta_0$ can be calculated before encoding. The classical data structure of $\Omega(c[i])$ can be constructed using classical method before encoding (see figure 3).

## II. QUANTUM VQ INEQUALITY ITERATION AND QUANTUM VQ ENCODING

Pang et.al., present a quantum representation of image to which database technique is applied [9], and this quantum method is also used to represent data of image in this paper.

**Definition 7** *Oracle $O_d$ :*

$$|\delta\rangle\,|x\rangle\,|i\rangle\,|c[i]\rangle\,|0\rangle \xrightarrow{O_d} |\delta\rangle\,|x\rangle\,|i\rangle\,|c[i]\rangle\,|d(x, c[i])\rangle$$

$O_d$ is used to compute the distance $d(x, c[i])$ and $(O_d)^{-1}$ is the inverse transformation of it.

**Definition 8** *Oracle $O_f$ :*

$$|\delta\rangle\,|x\rangle\,|i\rangle\,|c[i]\rangle\,|d(x, c[i])\rangle \xrightarrow{O_f} (-1)^{f(i)}\,|\delta\rangle\,|x\rangle\,|i\rangle\,|c[i]\rangle\,|d(x, c[i])\rangle$$

, where function $f(i)$ is defined as

$$f(i) = \begin{cases} 1 & \text{if } d(x, c[i]) < \delta \\ 0 & \text{otherwise} \end{cases}$$

**Definition 9** *quantum VQ inequality iteration $G_{inequality}$ :*

$$G_{inequality} = (2\,|\xi\rangle\,\langle\xi| - I)(U_L)^{-1}(O_d)^{-1}O_f O_d U_L$$

, where $|\xi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle$ and the unitary operation $U_L$ and $(U_L)^{-1}$ is $LOAD$ operation that loads data into registers from memory [9].

**Definition 10** *operation $U_L$:*

$$|\delta\rangle\,|x\rangle\,|i\rangle\,|0\rangle\,|0\rangle \xrightarrow{U_L} |\delta\rangle\,|x\rangle\,|i\rangle\,|c[i]\rangle\,|0\rangle$$

### A. Sub-procedure 1: the Quantum VQ for $x \in S$

**Algorithm 11** *Sub-procedure 1*

Step 1 Let $\delta = \frac{\delta_0}{2}$.

Step 2 Generate the initial state$| \ \Psi_0 >= \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} (|\delta\rangle \, |x\rangle \, |i\rangle \, |0\rangle \, |0\rangle)$.

Step 3 Perform the iteration $G_{inequality}$ $t = \left[\frac{\pi}{4}\sqrt{N}\right]$ times:

$$| \ \Psi_t >= (G_{inequality})^t \ | \ \psi_0 >$$

Step 4 Observe the third register: let $i_0$ be the outcome. Compute $d_0 = d(x, c[i_0])$ classically.

If $d_0 < \delta$ , the result is $i_0$ , otherwise go to Sub-procedure 2.

If there exists a vector $c[i_0] \in C$ such that $d(x, c[i_0]) < \frac{\delta_0}{2}$, the vector $c[i_0]$ is the closest codevector of $x \in S$ and it is the *unique* closest codevector [1]. And the iteration $G_{inequality}$ acts only on $N-$ dimensional subspace [9, 11]. Therefore, by Grover's algorithm, sub-procedure 1 performs the operation $G_{inequality}$ $\left[\frac{\pi}{4}\sqrt{N}\right]$ times approximately. That is, sub-procedure 1 has time complexity $\left[\frac{\pi}{4}\sqrt{N}\right]$.

### B. Sub-procedure 2: the Quantum VQ for $x \in (T - S)$ or $x \in (I - T)$

**Algorithm 12** *Sub-Procedure 2*

Step 1 Initialize $m = 1$ and $\lambda = \frac{6}{5}$.

Step 2 We choose an experiential value $\hat{\delta}$ such that the set $T$ can cover the whole encoding space $I$ approximately.

Step 3 Choose $j$ uniformly at random among the nonnegative integers not bigger than $m$.

Step 4 Generate the initial state $| \ \Psi_0 >= \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} (|\delta\rangle \, |x\rangle \, |i\rangle \, |0\rangle \, |0\rangle)$

Step 5 Apply $j$ iterations of the iteration $G_{inequality}$ starting from the state $| \ \Psi_0 >$:

$$| \ \Psi_j >= (G_{inequality})^j \ | \ \psi_0 >$$

Step 6 Observe the third register: let $h$ be the outcome.

Step 7 Perform classical computing $y_0 = d(x, c[h])$.

If $y_0 < \hat{\delta}$, classically full search the corresponding neighborhood $\Omega(c[h])$ to obtain the closest codevector $c[i_0]$ ; The result is $i_0$ and **exit**.

Step 8 Otherwise, set $m$ to $\min\{\lambda m, \sqrt{N}\}$ and go to step 3.

Step 9 **Encoding for $I-T$ :** If $y_0 \geq \hat{\delta}$ or $i_0$ does not exist, apply classical method to encode.

We have conclusion that the closest codevector $c[i_0]$ of vector $x \in T-S$ is included in the neighborhood $\Omega(c[h])$ of step 7 (see figure 1). The reason is that $d(x, c[j]) \geq d(c[j], c[h]) - d(c[h], x) > \hat{\delta}$ for an arbitrary $c[j] \notin \Omega(c[h])$.

The hybrid idea of sub-procedure 2 is that applying the iteration $G_{inequality}$ repeatedly to find a neighborhood $\Omega(c[h])$ which comprises very few elements, then full search the neighborhood classically to find the solution.

In addition, the set $I-T$ comprises very few vectors statistically and classical method is applied to encode vector $x \in I-T$.

Figure 1 illustrates the hybrid idea of sub-procedure 2.

# Insert Figure 1 Here

The iteration $G_{inequality}$ is embedded in the BBHT algorithm in sub-procedure 2. The case of no solution is handled by BBHT algorithm [8]. The iteration $G_{inequality}$ acts only on $N-$ dimensional subspace [9, 11]. Thus, the number of performing the iteration $G_{inequality}$ in sub-procedure 2 is not bigger than $\frac{9}{4}\sqrt{\frac{N}{Inf_\Omega}}$ when $Inf_\Omega \ll N$.

The four statistical properties of image for VQ are listed below (see figure 2):

Property 1 Almost feature vectors $x$ concentrate on themselves centroids generally. That is, almost vectors $x \in I$ are included in set $S$. The solution is *unique* for $x \in S$.

Property 2 The set $T-S$ comprises the points of near edges of image in practice. The solution is included in a small neighborhood $\Omega(c[h])$ for input vector $x \in T-S$.

Property 3 The set $I-T$ comprises special points such as very detail points or points at edges maybe.

Property 4 Let $\mid S \mid : \mid T-S \mid : \mid I-T \mid = a : b : c$. Statistically, we have (see figure 2)

$$80\% \leq a \leq 99\%,\ 1\% \leq b \leq 19\%, \text{ and } c < 1\%$$

Therefore, the time complexity of the whole quantum encoding algorithm is less than $\sqrt{N}$ statistically.

The phenomenon should be noticed, that the above properties are *not* powerful for classical methods to solve the open problem of the fast encoding of VQ, by contrast, it *is* powerful for quantum methods.

Figure 2 shows these statistical properties of image.

# Insert Figure 2 Here

The hybrid algorithm is not only an image compression method, but also it can be applied to image recognition [12].

## III. THE COST PHYSICAL RESOURCE FOR ACCESSORIAL DATA STRUCTURE

The main accessorial resource is cost to save the classical data structure of all neighborhoods $\Omega(c[i])$. And the data structure can be constructed using adjacency list technique [13] before encoding. Adjacency list technique is a basic technique, and it is applied on image compression usually [1] . The adjacency list to save all $\Omega(c[i])$ can be constructed by the following method [13]:

First, Allocate a single list to save all neighbors of each codevector $c[i]$, where $c[i] \in C$(i.e., centroid in figure 2) and $i = 0, 1, ..., (N-1)$.

Second, All of the addresses of all single lists are saved in an array.

The structure of the adjacency list is diagrammatized in figure 3.

It is easy to estimate the space complexity of the adjacency list[13]. The total number of the classical bits required to save the adjacency list is approximately $\sum_{i=0}^{N-1}(\mid \Omega(c[i]) \mid +1)(\log_2 N + 4 \times 8)$ (bits). In general, $\mid \Omega(c[i]) \mid$ is very small (see figure 2).

Therefore, the space complexity is $O(N \log_2 N)$ bits approximately and it is very good.

It's very easy to realize the adjacency list for modern electronic computer [13]. Therefore, it's a good tradeoff that costing $O(N \log_2 N)$ classical bits to permute the running time by decreasing 45 factor than the pure quantum algorithm presented in Ref.[11].

# Insert Figure 3 Here

## IV. CONCLUSION

VQ has high compression ratio and simple structure. However, the performance comes at the cost of increased computational complexity. Fast encoding of VQ is an open problem and is one of two key techniques of VQ. Many classical encoding algorithms of VQ that can obtain global optimal solution have computational complexity $O(N)$. In this paper, a hybrid quantum VQ encoding algorithm between classical method and quantum algorithm is presented. The number of its operations is less than $\sqrt{N}$ for most images. It's a good tradeoff that costing $O(N \log_2 N)$ classical bits to permute the running time by decreasing 45 factor than the pure quantum algorithm in Ref.[11].

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## V. FIGURES

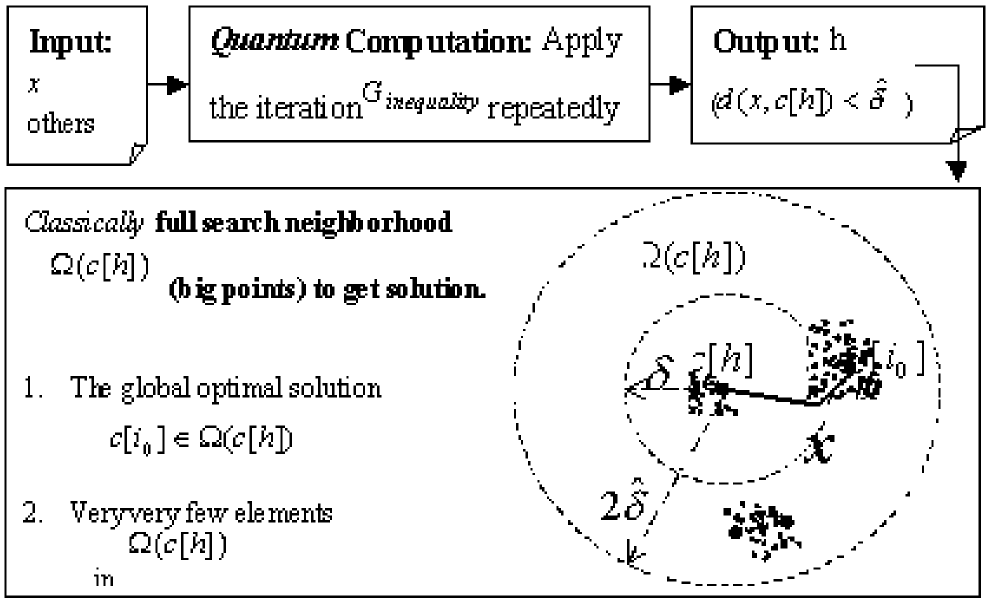


FIG. 1: The illustration of the hybrid idea of sub-procedure 2:
1. Embed the iteration $G_{inequality}$ in BBHT algorithm and apply the iteration repeatedly. This quantum method will find a neighborhood $\Omega(c[h])$.
2. The global optimal solution $c[i_0] \in \Omega(c[h])$, and there are very few elements in $\Omega(c[h])$. Therefore, we can classically search $\Omega(c[h])$ to obtain solution.

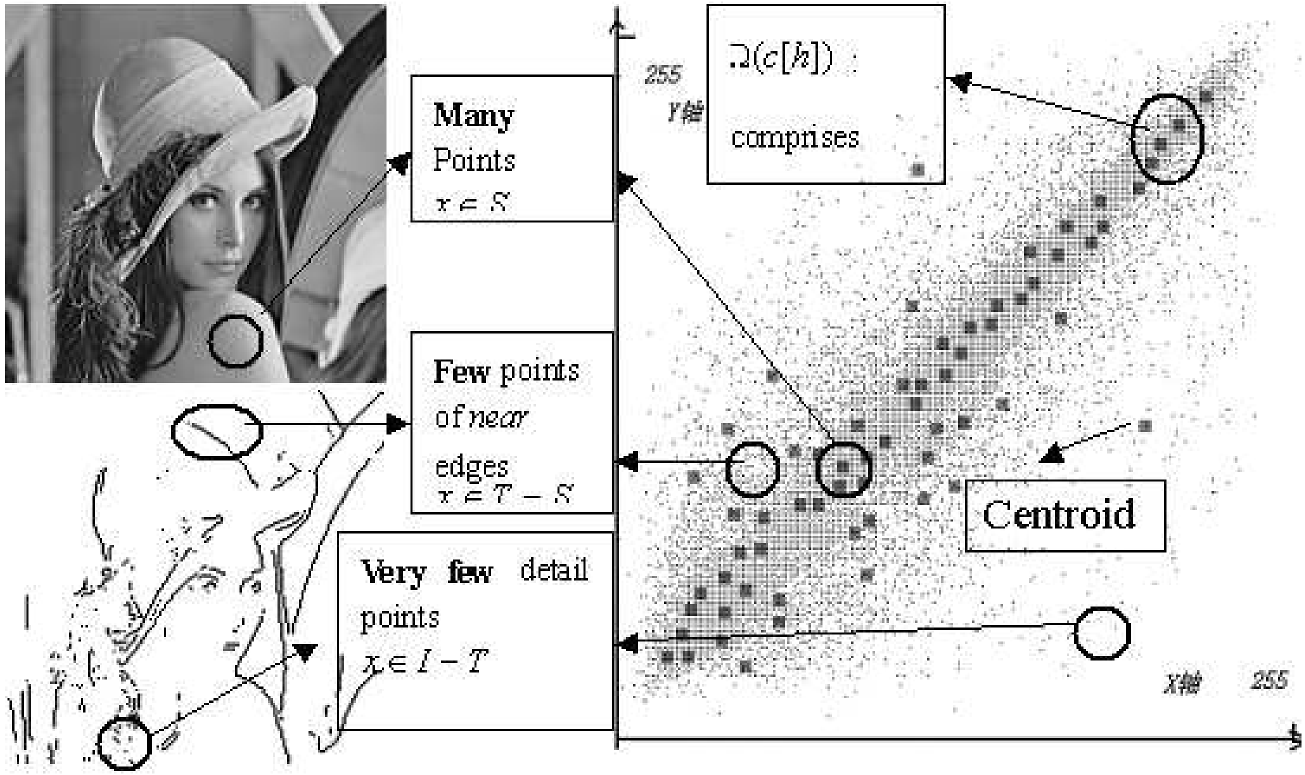


FIG. 2: The illustration of the properties of image that is applied to design the hybrid algorithm: (Image Lena with size256 $\times$ 256 is divided into $2 \times 1$ image blocks to form vectors [1].)
1. Almost feature vectors $x$ concentrate on themselves centroids generally. And the solution is unique for $x \in S$
2. The set $T - S$ comprises the points of near edges of image and it includes few points in practice. And the solution is included in a small set $\Omega(c[h])$for $x \in T - S$.
3. The set $I - T$ comprises special points such as very detail points or points at edges maybe.
4. Statistically, $\mid S \mid >> \mid T - S \mid >>> \mid I - T \mid$
The above four statistical properties of image for VQ are applied to accelerate the quantum algorithm:
1. Sub-procedure 1 acts on the set $S$ with time complexity $\left[\frac{\pi}{4}\sqrt{N}\right]$ approximately.
2. Sub-procedure 2 acts on the set $T - S$ with time complexity less than $\sqrt{N}$ approximately.
3. Classical full search algorithm acts on $I - T$.
The phenomenon should be noticed, that these properties are not powerful for classical methods to solve the open problem of the fast encoding of VQ, by contrast, it is powerful for quantum methods.

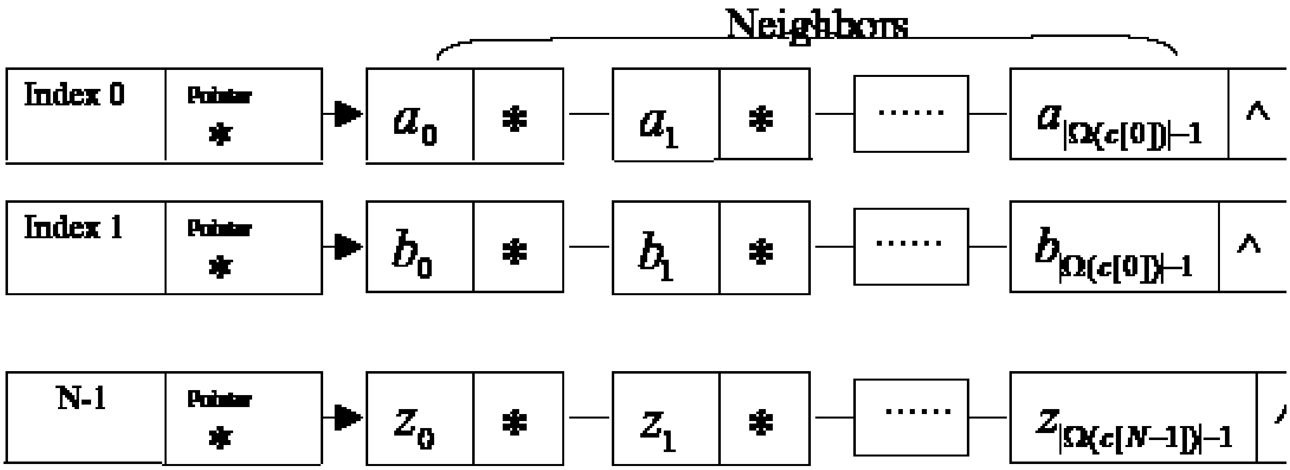


FIG. 3: Fig. 3 The illustration of the adjacency list to save all neighborhoods $\Omega(c[i])$: Each single list saves all neighbors of $c[i]$. The space complexity of the adjacency list is $O(N \log_2 N)$ and it is very good. It's very easy to realize the adjacency list for modern electronic computer. Therefore, it's a good tradeoff that costing $O(N \log_2 N)$ classical bits to permute the running time by decreasing 45 factor than the pure quantum algorithm in Ref.[11].